# Tutorial LTL CTL

## Question 1

1. $GFm$ {liveliness} model satisfies
2. $FGc$ {safety} model fails
3. $G[c \rightarrow X[Fm \wedge (\neg m\, U g)]]$ {safety} model fails

## Question 2

1. $EFAG\,\bar{a}$ [holds on none of the states]
2. $EG\,\bar{t}$ [holds on m and h]
3. $E(\bar{a}\,U\,t)$ [holds on h, c, g]
4. $AG(a \vee EG(\neg t))$ [does not hold on any state]
5. $EG[a \vee EG[\neg t]]$ [only c does not satisfy]
6. $AGAF(a \vee \neg t)$ [holds on all paths]

## Question 3

1. $A \wedge G[A \rightarrow XB \wedge XXB]$ $\Sigma = \{A, B, AB, \phi\}$

   ω-RE – $[AB]^\omega$

2. $G(A \rightarrow X((\neg A\, U(B \wedge \neg A)) \vee G\neg A)$ $\Sigma = \{A, B, C, AB, AC, BC, ABC, \phi\}$

   ω-RE : $(B+C)^*[A\cdot(B+C)^* B(B+C)^* + B + C]^\omega$

3. $G(XA \wedge XXB \rightarrow C)$

   ω-RE: $(B + C + A + C + (AB))^\omega + (BA)^\omega$

4. $G[\{(\neg A \wedge B)\, U\, C\} \Rightarrow XF(A \cap B)]$ $AP = \{A, B, C\}$

   ω-RE : $\{(A+B)^* + [(A+B)^* B^* + T^* X]\}^\omega$ $\Sigma = 2^{AP} = \{A, B, C, AB, BC, AC, ABC, \phi\}$

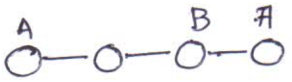
   $\alpha \subseteq \Sigma = \{AB, X\}$ where $X = AB$

Question 4 Equivalence checking

① $G(A \rightarrow B) \equiv GA \rightarrow GB$

not equivalent

counter example



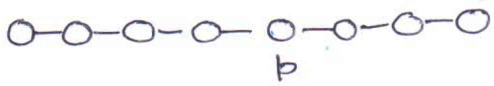

↳ L.H.S fails

R.H.S passes ( vacously true)

② $GFp \rightarrow GFq \equiv G(p \rightarrow Fq)$

not equivalent

counterexample:




p only occurs once in the trace $\pi$ and q never occurs

GFp is false so L.H.S is vacously true

When p occurs Fq does not occurs so R.H.S is false

3. $FGp \wedge FGq \equiv FG(p \wedge q)$

equivalent. Let path $\pi = s_0 s_1 s_2 \ldots$

Part 1: LHS $\rightarrow$ R.H.S

$\pi \models FGp \wedge FGq$

$\Rightarrow (\exists a \geq 0, \forall i \geq a \; s_i \models p) \wedge (\exists b \geq 0. \forall j \geq b, s_j \models q)$

Case 1 $a \geq b$

$\forall i \geq a \quad s_i \models p \;\&\; s_i \models q$

$\Rightarrow \forall i \geq a \quad s_i \models (p \wedge q)$

$\Rightarrow \pi \models FG(p \wedge q)$

Case 2 $b \geq a$

$\forall i \geq b \quad s_i \models p \;\&\; s_i \models q$

$\Rightarrow \forall i \geq b \quad s_i \models (p \wedge q)$

$\Rightarrow \pi \models FG(p \wedge q)$

$\therefore$ if $\pi \models (FGp \wedge FGq)$, then $\pi \models FG(p \wedge q)$ — 1

Part 2: R.H.S $\rightarrow$ L.H.S

$\pi \models FG(p \wedge q)$

$\Rightarrow \exists a \geq 0, \forall i \geq a \; s_i \models (p \wedge q)$

$\Rightarrow \exists a \geq 0, \forall i \geq a \quad s_i \models p \;\&\; s_i \models q$

$\because a \geq 0, \; s_0 \models FGp$ and $s_0 \models FGq$

$\Rightarrow s_0 \models (FGp \wedge FGq) \Rightarrow \pi \models (FGp \wedge FGq)$

$\therefore$ if $\pi \models FG(p \wedge q)$, then $\pi \models (FGp \wedge FGq)$ — 2

From 1 & 2 $\quad FGp \wedge FGq \equiv FG(p \wedge q)$